On perspectives and trends in model theory through one (aging) individual's looking glass

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Recent Developments in Model Theory
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No Predictions

"Allow me two anecdotes. In about 1970 a Polish logician reported that a senior colleague of his had advised him not to publish a textbook on first-order model theory, because the subject was dead. And in 1966 David Park, who had just completed a PhD in first-order model theory with Hartley Rogers at MIT, visited the research group in Oxford and urged us to get out of first-order model theory because it no longer had any interesting questions."

W. Hodges, *Model theory*

A Theme

"I present the material from the point of view of someone who has been in the subject for forty years and who has seen ideas come and go. I urge the younger participants to ponder Lang's statements (a propos algebraic number theory):

'It seems that, over the years, everything that has been done has proved useful, theoretically or as examples, for the further development of the theory. Old and seemingly isolated special cases have continuously acquired new significance, often after half a century or more.'
S. Lang, forward to *Algebraic number theory* "

A. Macintyre, *A history of the interactions between logic and number theory*

Several good historically based articles/talks

- Hodges, *Model Theory*

- Macintyre, *A history of the interactions between logic and number theory*

- Vaught, *Model theory before 1945* (Tarski Symposium volume)

- Chang, *Model theory 1945-1971* (Tarski Symposium volume)

- Kolaitis, *Reflections on finite model theory* (slides from 2007 LICS talk)

Very early work: 1930's and before

- Tarski

- Gödel

- Mal'cev

To the end of the 1950's (roughly)

- Julia Robinson

- Tarski's QE for (first-order) theory of real field appears

- Finite model theory: Trakhtenbrot, Spectrum Problem

- Model theory emerges as a subject

- Preservation theorems—the infinite and finite (failure of substructure preservation in finite)

To the end of the 1950's (roughly), cont'd

- Berkeley school

- Fraïssé

- Łos Conjecture

- Feferman-Vaught

- Abraham Robinson—*Complete theories*, nonstandard analysis

1960's (roughly)

- Hilbert's 10th Problem

- Morley-Vaught; Vaught's conjecture

- Ax-Kochen, Ershov 1965, 66

- Morley, *Categoricity in power*, 1965

1960's (roughly), cont'd

- Model theory and set theory

- Ax, finite fields, 1968

- Beyond first-order logic: infinitary logics (admissible sets), $L(Q)$

- Lindstrom's Theorem

1970's (roughly)

- SHELAH

- Sacks, *Saturated Model Theory* "This book was written in answer to one question: 'Does a recursion theorist dare to write a book on model theory?' "

- Baldwin-Lachlan Theorem

- Chang-Keisler published

1970's (roughly), cont'd

- $0 - 1$ Law for finite relational structures

- Collins: cylindrical algebraic decomposition

- NP=$\exists$ second order

- S is a spectrum $\Leftrightarrow$ S is in NEXPTIME. (So get equivalence of closure under complement)

- Relational databases

1970's (roughly), cont'd

- DCF

- Macintyre QE for $\mathbb{Q}_p$

- Lascar-Poizat version of forking

- Stability and algebra

- Cherlin-Zilber conjecture

1970's (roughly), cont'd

- Publication of Classification Theory and the Number of Non-isomorphic Models (first ed.)

1980's (roughly)

- Totally categorical theories are not finitely axiomatizable

- Cherlin-Harrington-Lachlin ($\aleph_0$-stable, $\aleph_0$-categorical)

- Zilber Trichotomy Conjecture

- Immerman-Vardi: For ordered finite structures, P=LFP (first-order and least fixed point on positive FO formulas).

- Pillay, *An introduction to stability theory*

1980's (roughly), cont'd

- Forking Festivals, ultimately Baldwin's *Fundamentals of stability theory*

- O-minimality

- Hrushovski

- Geometric stability theory

- Poizat, *Groupes Stables*

1980's (roughly), cont'd

- Borovik program

- Denef and Denef-van den Dries: rationality of Poincaré series

- Independence property and VC dimension

- Hrushovski's constructions

1990's (roughly)

- Zariski Geometries

- Mordell-Lang

- ACFA

- Manin-Mumford

1990's (roughly), cont'd

- Definable sets in finite fields: Chatzidakis-van den Dries-Macintyre

- Wilkie: Model completeness and o-minimality of $(\bar{\mathbb{R}}, \exp)$

- van den Dries-Macintyre-Marker $(\mathbb{R}_{an,exp})$

- Pillay, *Geometric stability theory*

1990's (roughly), cont'd

- Simple theories

- Wilkie, o-minimality of expansion of $\bar{\mathbb{R}}$ by Pfaffian functions

- Peterzil-Starchenko o-minimal trichotomy theorem

- Peterzil-Pillay-Starchenko, a version of Cherlin-Zilber in o-minimal setting

- Shelah-Spencer & Baldwin Shelah

2000's (roughly)

- Rossman, preservation for finites under homomorphism

- This Meeting!