# $\mathbb{Q}$ACFA

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Recent Developments in Model Theory
Oléron, France
June 7, 2010

# Many small signatures: first observations

- Signatures $L_1 \subset L_2 \subset \ldots$ and $L := \cup_n L_n$ (or other directed system). If $L' \subset L$ is finite, $L' \subset L_n$ for some $n$.
- $L_n$-theories $T_n$ such that $T_n \subset T_{n+1}$, $T := \cup_n T_n$ is consistent, and $T_n$ contains the reduct of $T_{n+1}$ to $L_n$.

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Recall: $L_1 \subset L_2 \subset \ldots \subset L := \cup_n L_n$ and $T := \cup_n T_n$.

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  If for some $N \in \omega$, $U_n(p_n) \leq N$ for all $n$, then $U(p) \leq N$.

# Attributions

Pure model theory, ACF: lore and exercises.

ACFA: Chatzidakis, Hrushovski, Peterzil.

$\mathbb{Q}$ACFA: Me.

# Please meet $\mathbb{Q}$ACFA:

- $L$ is the signature $L_{Rings} \cup \{\sigma_q \mid q \in \mathbb{Q}\}$.
  $S$ is the theory of fields with a $(\mathbb{Q}, +)$ action:
  each $\sigma_q$ is a field automorphism and $\sigma_q \circ \sigma_r = \sigma_{q+r}$.

# Please meet $\mathbb{Q}$ACFA:

- $L := L_R \cup \{\sigma_q \mid q \in \mathbb{Q}\}$, $S := Th$(fields with $(\mathbb{Q}, +)$-action): each $\sigma_q$ is a field automorphism and $\sigma_q \circ \sigma_r = \sigma_{q+r}$.
- $L_n := L_R \cup \{\sigma_{\frac{m}{n!}} \mid m \in \mathbb{Z}\}$ and $S_n := S|_{L_n}$. Note $S_n \subset S_{n+1}$.

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- Let $T_n := ((K, \sigma_{\frac{1}{n!}}) \models \text{ACFA})$.
- If $(K, \tau) \models$ ACFA, then $(K, \tau^n) \models$ ACFA. So $T := \cup_n T_n$ is a consistent $L$-theory: $\mathbb{Q}$ACFA.

# From ACF to ACFA to $\mathbb{Q}$ACFA.

Let $A, B, C \subset K \models ACF$ and $\mathrm{acl} := \mathrm{acl}_{fields}$, and let $\mathop{\smile\hspace{-0.9em}\mid\hspace{0.6em}}$ be algebraic independence.

Let $(K, \sigma) \models$ ACFA, and $\langle A \rangle_{\mathrm{ACFA}} := \cup_{n \in \mathbb{Z}} \sigma^n(A)$

- Model-theoretic algebraic closure is

$$\mathrm{acl}_{\mathrm{ACFA}}(A) := \mathrm{acl}(\langle A \rangle_{\mathrm{ACFA}})$$

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- $F_1 \gneq F_{\frac{1}{2}} \gneq F_{\frac{1}{6}} \gneq F_{\frac{1}{24}} \dots$ is an infinite descending chain of definable fields: this breaks *super*simplicity and quantifier-free-*super*stability.

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- What is the induced structure on these $F_q$? On $\cup_{q\in\mathbb{Q}} F_q$? Something like separably closed fields, or PAC fields, or?

# Away from fixed fields: finite rank

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- The type $p$ is orthogonal to fixed fields iff all $p_n$ are.
- Usually (and probably always), if $(A, B)^{\sigma_1 \sharp}$ is a minimal modular group in the reduct to $L_1$, then it is finite rank modular in $L$.